Quasiparticles in a thermal process

Ferenc Márkus[1,*] and Katalin Gambár[2,†]
[1]*Institute of Physics, Budapest University of Technology and Economics, Budafoki út 8, H-1521 Budapest, Hungary*
[2]*Department of Atomic Physics, Roland Eötvös University, Pázmány Péter sétány 1/A, H-1117 Budapest, Hungary*
(Received 24 January 2005; published 17 June 2005)

We introduce an abstract scalar field and a covariant field equation, by which we make an attempt to connect the Fourier heat conduction and wavelike heat propagation. This field can be the generalization of the usual temperature from a dynamical point of view. It is shown that a kind of effective mass of this thermal process can be calculated. Finally, we express the unit of dissipative action with the help of universal constants.

DOI: 10.1103/PhysRevE.71.066117 PACS number(s): 05.70.Ln, 04.20.Fy, 11.10.Ef

The transport phenomena mean in general dissipative processes, and their description is based on the methods of classical field theory [1,2]. We managed to give the Lagrange density function [3] for those field equations that are given by linear parabolic differential equations [4–6]. One of the most important cases is the temperature field in local equilibrium. The process occurs in this field governed by the parabolic differential equation, the Fourier heat equation,

$$\frac{\partial \mathcal{T}}{\partial t} - \frac{\lambda}{c_v}\frac{\partial^2 \mathcal{T}}{\partial x^2} = 0, \tag{1}$$

where $\mathcal{T}$ is the temperature, λ is the heat conductivity, and c_v is the specific heat capacity (referring to unit volume). Of course, the hypothesis of local equilibrium restricts the speed of processes, i.e., this is valid for those kinds of fields where the processes themselves are very slow.

To apply the Hamilton-Lagrange formalism for parabolic transport equations like heat conduction, we have worked out a potential-based formulation [5,6]. In this description we can write the Lagrange density function for this dissipative process as

$$L = \frac{1}{2}\left(\frac{\partial \varphi}{\partial t}\right)^2 + \frac{1}{2}\frac{\lambda^2}{c_v^2}\left(\frac{\partial^2 \varphi}{\partial x^2}\right)^2, \tag{2}$$

where we introduce a scalar (potential) field $\varphi(x,t)$. This generates the measurable temperature field $\mathcal{T}(x,t)$,

$$\mathcal{T}(x,t) = -\frac{\partial \varphi}{\partial t} - \frac{\lambda}{c_v}\frac{\partial^2 \varphi}{\partial x^2}, \tag{3}$$

and this is the definition of the potential field $\varphi(x,t)$ [5,6] at the same time.

Moreover, in this way we have the tools for the canonical quantization of a dissipative process. We have introduced the canonically conjugated quantities and we have expressed the Hamiltonian of the field. The commutation rules are

$$[P_k^{(C)}, C_l] = h\delta_{kl}, \tag{4}$$

$$[P_k^{(S)}, S_l] = h\delta_{kl}, \tag{5}$$

where C_l and S_l are the operators coming from the Fourier coefficients of $\varphi(x,t)$. $P_k^{(C)}$ and $P_k^{(S)}$ are the conjugated momenta and the constant h is the unit of dissipative action. We calculate h in terms of universal constants at the end of this paper.

Introducing the temperature operator [7,8], we can obtain its eigenvalues $(\lambda/c_v)k^2h$, where k is the wave number. The eigenvalue of energy is $\varepsilon(k)=\lambda k^2 h$.

We try to make a step toward the description of the kind of heat processes, in which the speed of heat propagation compared with the speed of action cannot be neglected [4,9–13]. Let us take the following invariant Lagrange density function:

$$L = \frac{1}{2}(\partial_{\mu\nu}\varphi)(\partial^{\mu\nu}\varphi) - \frac{1}{2}\frac{c^4 c_v^4}{16\lambda^4}\varphi^2, \tag{6}$$

where c is the speed of light. $x^\mu=(x^0,x^1,x^2,x^3)=(ct,x,y,z)$ is a contravariant four-vector, $x_\mu=(x_0,x_1,x_2,x_3)=(ct,-x,-y,-z)$ is a covariant four-vector, $\partial_\mu=\partial/\partial x^\mu$, and $\partial^\mu=\partial/\partial x_\mu$. Thus, the covariant Euler-Lagrange equation as the equation of motion can be written

$$\partial_\mu\partial_\nu\partial^\mu\partial^\nu\varphi - \frac{c^4 c_v^4}{16\lambda^4}\varphi = 0. \tag{7}$$

We calculate it for the one-dimensional case

$$\frac{\partial^4\varphi}{c^4\,\partial t^4} + \frac{\partial^4\varphi}{\partial x^4} - 2\frac{\partial^4\varphi}{c^2\,\partial t^2\,\partial x^2} - \frac{c^4 c_v^4}{16\lambda^4}\varphi = 0. \tag{8}$$

Let us define an abstract field quantity T by the potential function φ

$$T = \partial_\mu\partial^\mu\varphi + \frac{c^2 c_v^2}{4\lambda^2}\varphi. \tag{9}$$

It is easy to check that T is an invariant scalar and the field equation for T is

*Corresponding author. Email address: markus@phy.bme.hu; markusferi@tvn.hu
†Email address: gambar@ludens.elte.hu; gakat@tvn.hu

$$\partial_\mu \partial^\mu T - \frac{c^2 c_v^2}{4\lambda^2} T = 0, \tag{10}$$

and for the one-dimensional case we can write

$$\frac{1}{c^2}\frac{\partial^2 T}{\partial t^2} - \frac{\partial^2 T}{\partial x^2} - \frac{c^2 c_v^2}{4\lambda^2} T = 0. \tag{11}$$

Expressing the dispersion relation of the problem we obtain

$$\omega_T(c) = \sqrt{c^2 k^2 - \frac{c^4 c_v^2}{4\lambda^2}}. \tag{12}$$

Since we demand causality (the theory is linear); thus the value

$$k_0 = \frac{c c_v}{2\lambda} \tag{13}$$

means a physical limit. If $k > k_0$ than the process can be described with a temperature wave (a wavelike heat propagation), whose speed can be expressed

$$w = \frac{\omega}{k} = c\sqrt{1 - \frac{c^2 c_v^2}{4\lambda^2 k^2}} < c. \tag{14}$$

If $k \ll k_0$ the theory should provide the classical (long wavelength) behavior, i.e., the group velocity $(d\omega/dk)$ should be the speed of heat propagation in the case of Fourier heat conduction. This relation can be obtained from Eq. (1). The dispersion relation is

$$i\omega c_v - \lambda k^2 = 0, \tag{15}$$

by which we can write the group velocity

$$\frac{d\omega}{dk} = -i\frac{2\lambda}{c_v}k. \tag{16}$$

The complex unit i shows that the heat conduction is not a wave propagation. We can compare this result with the wavelike solution if we consider Eq. (12), we calculate the dispersion relation and we take the limit $c \to \infty$. We can immediately see that we can get back the classical limit

$$\frac{d\omega}{dk} = \frac{1}{\sqrt{1/c^2 - c_v^2/4\lambda^2 k^2}} \approx -i\frac{2\lambda}{c_v}k. \tag{17}$$

This means that the propagation of the thermal effects is much slower than the speed of light

$$v_{\mathcal{T}} = \frac{2\lambda}{c_v}k \ll c. \tag{18}$$

Here, we considered the classical limit $k \ll k_0$ and $c \gg 1$.

It seems to us that k_0 differentiates two different behaviors of the propagation of heat. When $k > k_0$ the heat propagates as a wave, i.e., there is no dissipation and the speed w is a large value. When $k < k_0$ the temperature propagation is the classical heat conduction), i.e., the process is dissipative and the speed $v_{\mathcal{T}}$ is small.

Previously, we have pointed out the possibility of existence of quasiparticles [14] in the heat conduction [7,8], and now we show a different way to predict these excitations. Here, we mention that there are several papers that deal with the existence of thermal particles from different viewpoints [15–18]. For this reason, we turn back to the examination of Eq. (8) and we write it again as

$$\frac{\partial^4 \varphi}{c^4\,\partial t^4} + \frac{\partial^4 \varphi}{\partial x^4} - 2\frac{\partial^4 \varphi}{c^2\,\partial t^2\,\partial x^2} - \frac{c^4 c_v^4}{16\lambda^4}\varphi = 0. \tag{19}$$

This equation can be considered as a one-particle wave equation. We realize that this may come from that equation where we used the usual operator calculus, i.e., we have substituted the energy operator

$$E = -\frac{h_p}{i}\frac{\partial}{\partial t} \tag{20}$$

and the momentum operator

$$p = \frac{h_p}{i}\frac{\partial}{\partial x}. \tag{21}$$

h_p is the Planck constant per 2π. The obtained one-particle equation is

$$\frac{E^4}{c^4 h_p^4} + \frac{p^4}{h_p^4} - \frac{2E^2 p^2}{c^2 h_p^4} - \frac{c^4 c_v^4}{16\lambda^4} = 0, \tag{22}$$

which can be written as a product

$$\left(E^2 - p^2 c^2 - \frac{c^4 c_v^2}{4\lambda^2}h_p^2\right)\left(E^2 - p^2 c^2 + \frac{c^4 c_v^2}{4\lambda^2}h_p^2\right) = 0. \tag{23}$$

The first multiplier pertains to a positive mass particle

$$E^2 = p^2 c^2 + \frac{c^4 c_v^2}{4\lambda^2}h_p^2; \tag{24}$$

the second multiplier corresponds to an imaginary mass

$$E^2 = p^2 c^2 - \frac{c^4 c_v^2}{4\lambda^2}h_p^2. \tag{25}$$

However, it is not clear what an imaginary mass may mean. Here, the question is open. Now, let us consider Eq. (24), in which the second term on the right hand side should be the square of the self-energy $m_0^2 c^4 = (c^4 c_v^2/4\lambda^2)h_p^2$. We can read the rest mass m_0 of this quasiparticle as

$$m_0 = \frac{c_v}{2\lambda}h_p. \tag{26}$$

[A negative value of $m_0 = -(c_v/2\lambda)h_p$ also appears, but we do not understand what it may mean.] If we introduce the thermal diffusivity $D = \lambda/c_v$, we can write the mass $m_0 = h_p/2D$, and the self-energy of the quantum can be written $E_0 = h_p/2Dc^2$. We express the momentum $p = h_p k$, and we write the kinetic energy of the quasiparticle in the classical form $(p^2/2m_0)$. Here, we have substituted the rest mass m_0 by which the description turns to the classical theory [19], i.e.,

$$\epsilon = \frac{h_p^2 k^2}{2m_0}. \tag{27}$$

One can expect that the appearance of this massive quasiparticle does not mean a contradiction for the case of heat diffusion. We can show this if we substitute the m_0 from Eq. (26) by which we obtain

$$\epsilon = \frac{\lambda}{c_v} h_p k^2. \tag{28}$$

Furthermore, the kinetic energy E_k can be expressed by the mass m_0 and the classical group velocity of heat propagation $v_{\mathcal{T}}$ from Eq. (18),

$$E_k = \frac{1}{2} m_0 v_{\mathcal{T}}^2. \tag{29}$$

We take the expressions of $v_{\mathcal{T}}$ and m_0 from Eqs. (18) and (26), and we obtain

$$E_k = \frac{\lambda}{c_v} h_p k^2 = \frac{h_p^2 k^2}{2m_0} = \epsilon, \tag{30}$$

which is expected from a consistent theory.

We can calculate the average energy of one quantum if we assume a weight factor $w(k)$, which gives the distribution of the possible states as a function of wave number k. Now, we do not need to say anything about the weight function, we just formally express the average of ϵ,

$$\overline{\epsilon_1} = \frac{\int_0^\infty (\lambda/c_v) h_p k^2 w(k) dn(k)}{\int_0^\infty dn(k)} = \frac{\lambda}{c_v} h_p \frac{\int_0^\infty k^2 w(k) dn(k)}{\int_0^\infty dn(k)}, \tag{31}$$

where $dn(k)$ denotes the density of states. We can take into account that the specific heat of one degree of freedom is $c_v = k_B/2$, where k_B is the Boltzmann constant, so we obtain

$$\overline{\epsilon_1} = \frac{2\lambda}{k_B} h_p \frac{\int_0^\infty k^2 w(k) dn(k)}{\int_0^\infty dn(k)}. \tag{32}$$

On the other hand, we can calculate the average energy of one quantum when we use the expression $\varepsilon(k) = \lambda k^2 h$,

$$\overline{\epsilon_1} = \frac{\int_0^\infty \varepsilon(k) w(k) dn(k)}{\int_0^\infty dn(k)} = \lambda h \frac{\int_0^\infty w(k) dn(k)}{\int_0^\infty dn(k)}. \tag{33}$$

We expect that the values of these averages of quanta, which were derived by different ways [Eqs. (32) and (33)], although they are related to the same physical process, should be equal. We immediately realize that

$$h = \frac{2h_p}{k_B}, \tag{34}$$

i.e., the constant h (unit of dissipative action) can be traced back to the universal constants h_p and k_B. We can express the quantum of heat by these,

$$\varepsilon(k) = 2\lambda \frac{h_p}{k_B} k^2. \tag{35}$$

Summarizing the results, we can see that the quantum field theory of dissipative processes enables us to get a deeper insight into the physical processes. The nonequilibrium thermodynamics can handle the concept of irreversibility, and the field theory can lead us to the microscopic level. In the present work we elaborated the wavelike generalization of heat propagation, and we could calculate the effective mass of quasiparticles of a thermal process. The value of the effective mass depends on the thermal diffusivity and the Planck constant. The greater thermal diffusivity means a smaller mass of quasiparticles.

Katalin Gambár would like to thank the Bolyai János Research Foundation of the Hungarian Academy of Sciences for financial support.

[1] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1986).
[2] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, Cambridge, U.K., 1995).
[3] Ph. M. Morse and H. Feshbach, *Methods of Theoretical Physics I* (McGraw-Hill, New York, 1953).
[4] S. Sieniutycz, *Conservation Laws in Variational Thermo-Hydrodynamics* (Kluwer Academic, Dordrecht, 1994).
[5] K. Gambár and F. Márkus, Phys. Rev. E **50**, 1227 (1994).
[6] K. Gambár, K. Martinás, and F. Márkus, Phys. Rev. E **55**, 5581 (1997).
[7] K. Gambár and F. Márkus, Open Syst. Inf. Dyn. **8**, 369 (2001).
[8] F. Márkus and K. Gambár, Phys. Rev. E **54**, 4607 (1996).
[9] D. D. Joseph and L. Preziosi, Rev. Mod. Phys. **61**, 41 (1989); **62**, 375 (1990).
[10] W. Dreyer and H. Struchtrup, Continuum Mech. Thermodyn. **5**, 3 (1993).
[11] D. Jou, J. Casas-Vázquez, and G. Lebon, *Extended Irreversible Thermodynamics* (Springer, Berlin, 2003).
[12] I. Müller and T. Ruggeri, *Rational Extended Thermodynamics* (Springer, New York, 1997).
[13] J. Casas-Vázquez and D. Jou, Rep. Prog. Phys. **66**, 1937 (2003).
[14] J. M. Ziman, *Principles of the Theory of Solids* (Cambridge

University Press, Cambridge, U.K., 1972).
[15] A. I. Veinik, *Thermodynamics of Irreversible Processes* (Nauka i Technika, Minsk, 1966) (in Russian).
[16] S. Sieniutycz and R. S. Berry, Open Syst. Inf. Dyn. **4**, 15 (1997).
[17] K.-H. Anthony, J. Non-Newtonian Fluid Mech. **96**, 291 (2001).
[18] K.-H. Anthony, Acta Phys. Hung. **67**, 321 (1990).
[19] The classical approximation $(v \ll c)$ is based on the following calculation: $mc^2 \approx (m_0/\sqrt{1-v^2/c^2})c^2 \approx [m_0/(1-v^2/2c^2)]c^2 \approx m_0(1+v^2/2c^2)c^2 = (1/2)m_0 v^2 + m_0 c^2 = p^2/(2m_0) + m_0 c^2$.